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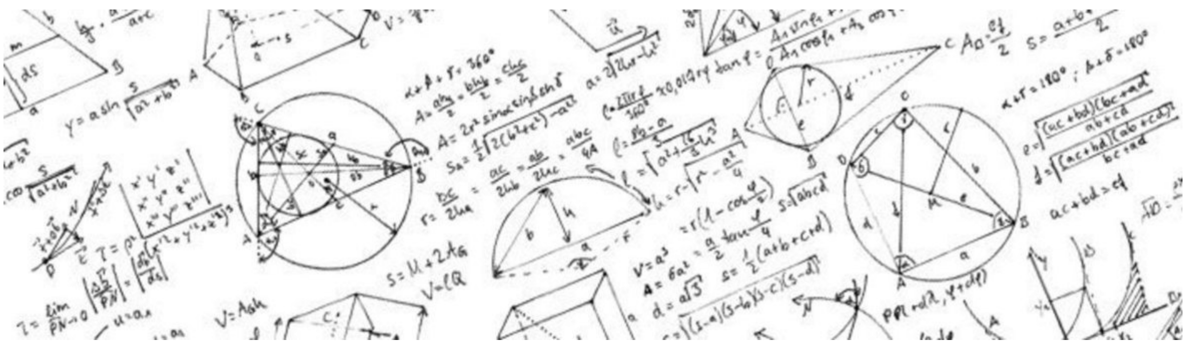
What's the Math and Stats Department Teaching Next Term?

Have you checked your registration time yet? Made a list of classes you're hoping to take next year? Let the course descriptions below guide you into an adventurous new term with the Carleton Department of Mathematics and Statistics! There's something for everybody, from mathematics of climate to applied regression. Find out more below.



Math Classes

Math 236: Mathematical Structures
Instructor: Claudio Gómez-González
Time: 2a, sophomore priority
Prerequisite: Math 232 or instructor permission

How do we prove mathematical statements? How do we even think of possible statements, and what makes us suspect that a particular statement may be true? There are no easy, general answers. Mathematics is a complex subject, with a great variety of living and growing branches, and with deep roots that tap into the wisdom of many generations. Still, if you've ever wondered "How did anyone come up with that?", or "How can you really be sure of that?", about some mathematical result, taking this course may help dispel some of the mystery. We'll explore various concepts, especially from set theory, that are indispensable for most areas of advanced mathematics, and we'll spend considerable time developing theorem-proving and problem-solving skills. Along the way we'll take a new and closer look at some old friends, such as functions and relations: What are they really? In the final part of the course we'll use functions to compare "sizes" of various infinite sets. For example, we'll see that despite appearances, there are not any "more" rational numbers than there are integers; on the other hand, there are "more" real numbers than rational numbers. If you're considering a math major, taking this course should help you decide; also, "Structures" is a prerequisite for the majority of upper-level math courses.

Math 241: Ordinary Differential Equations
Instructor: Rob Thompson
Time: 4a
Prerequisite: Math 232

Differential equations are a fundamental language used by mathematicians, scientists and engineers to understand and describe processes involving continuous change. In this course we will study differential equations from both a practical and theoretical point of view. Our focus will be on developing differential equation models from natural laws and exploring the mathematical ideas that arise within these models.

We'll study examples like mechanical vibrations, lasers, insect outbreaks, competition and cooperation of species, language coexistence, superconducting circuits, and much more! The science will stay at an elementary level; our focus will be the mathematical ideas that arise in these models. Feel free to contact Rob (rthompson) with any questions!

Math 244: Geometries
Instructor: MurphyKate Montee
Time: 2a
Prerequisite: Math 236

Geometry has been fascinating humans around the world for millennia. We use geometric patterns as decorations, we use projective geometry to create optical illusions, and geometric proofs and ideas have been used to illustrate the power and beauty of math. Come see what the hype is all about! We'll start with flat geometry (aka Euclidean geometry), and then move on to more surprising spaces like hyperbolic planes, where there are no rectangles, the sum of the interior angles in a triangle is always less than 180, and the area of any triangle is completely determined by the sum of its angles. It's weird, beautiful, and fun!

Math 295: Mathematics of Climate
Instructor: Kate Meyer
Time: 4/5c
Prerequisites: Math 120 or 211

What determines the Earth’s climate, and what aspects of climate change can we predict using models? This course will introduce students to mathematical methods for studying planetary climate. The focus will be on low-dimensional models, whose simplicity allows insight into fundamental mechanisms of climate change. We will use tools from algebra, geometry, and calculus to study topics including energy balance, greenhouse gas forcing, and ice-albedo feedback. This course will count towards the Applied Math area of the math major.

Math 321: Real Analysis I
Instructor: Alex Barrios
Time: 3a
Prerequisites: Mathematics 236 or instructor permission

How was calculus discovered? How does one prove statements in calculus, such as the Intermediate Value Theorem and the Fundamental Theorem of Calculus? These are questions that we will tackle as part of our course.

In the early 1600s, mathematicians reconsidered the following two questions from antiquity:
1. Given a geometric shape, what is its area?
2. Given a geometric shape, what is a tangent line to a point on the shape?

These two questions were revived with the introduction of Cartesian coordinates by René Descartes and Pierre de Fermat. In the decades that followed, mathematicians began to lay the foundations for what we call calculus today. Yet, a rigorous treatment of the subject would have to wait until the 1800s. In this course, we will build the theoretical foundations of calculus from the group up. Consequently, we will be able to rigorously prove familiar statements from single-variable calculus. This course is highly recommended for anyone considering grad school in math or statistics.

Math 341: Partial Differential Equations
Instructor: Rob Thompson
Time: 5a
Prerequisite: Math 241

About 200 years ago, Jean Baptiste Fourier studied the way that heat moves through a flat metal plate via a partial differential equation called the heat equation. Trying to describe his observations mathematically, he did a seemingly simple thing: he expressed the heat distribution as a sum of sines and cosines (a "Fourier series”). This simple idea revolutionized pure and applied mathematics!

In this course, we'll learn the fundamentals of partial differential equations and make a tour of Fourier's revolution. We'll examine various interesting PDE (including the heat equation) and their applications to wave propagation, heat conduction, elastic equilibrium, and more. We'll also develop ideas from Fourier analysis as needed to access information about the solutions to the PDE we study. Although these equations are motivated by scientific applications, our focus will be the mathematical ideas that arise from trying to understand and solve them. Feel free to contact Rob (rthompson) with any questions!

Math 352: Topics in Abstract Algebra
Instructor: Mark Krusemeyer
Time: 2a
Prerequisite: Math 342

So you liked Abstract Algebra I? Then it might well get even better, now that you have the tools to study a particular topic or two in some depth. This year's main topic is Galois theory. (There will be essentially no overlap between this class and last year's Math 352. We are offering Math 352 every year, but never repeating the same topic in consecutive years, so you can take "the" course - which will actually be two different courses - twice in a row.) Galois theory establishes an unexpected deep connection between fields and groups - more precisely, between field extensions and groups of automorphisms - and is used widely elsewhere in mathematics, especially within algebra. We will start by studying field extensions in general, and we'll soon see how the concept of dimension from linear algebra can be used to show that it is impossible to trisect an arbitrary angle using only straightedge and compasses (in other words, if you can only draw straight lines and circular arcs; note that some special angles, such as right angles, *can* be trisected).

Once we have some more "machinery", we'll be able to prove a beautiful result about the "Galois correspondence" (the connection between fields and groups mentioned above). Using this so-called Fundamental Theorem (have you noticed that several areas of mathematics have such a theorem?), we should be able to show one of the earliest and most striking applications of Galois theory: the result that quintic (degree 5) polynomial equations *cannot* be solved by using only arithmetic operations and taking radicals (that is, square roots, cube roots, fifth roots, etc.). Galois' proof of this wasn't published until 1846, fourteen years after his untimely death in a duel; a different proof had been found by Abel in 1823 after Ruffini had announced the result in 1799. Before that, ever since the Renaissance, mathematicians had been searching for a way to solve quintic equations using radicals. They were confident at first because, starting from the well-known quadratic formula, they had found such formulas to solve cubic and quartic equations, but then they came to realize that quintic equations seemed, somehow, to be essentially more difficult ...

Math 361: Complex Analysis
Instructor: Owen Biesel
Time: T/Th 10:10-11:55
Prerequisites: Mathematics 321 or instructor permission

The real numbers are to the complex numbers as real analysis is to complex analysis... but complex analysis is actually a nicer subject! Did you know that if a function is differentiable, then it is also infinitely differentiable? Did you know that if a function is bounded and differentiable everywhere then it must be constant? These facts aren't true for real-valued functions of a real variable, but they are true for complex-valued functions of a complex variable! In this class you'll learn about the amazing consequences of complex differentiability, the spectacular singularities a complex function can have, and why the integral of 1/(x^2+1) over the real line has anything to do with pi. (Spoiler: it's because of the way the function blows up at x=i!) This class is recommended for anyone who is considering grad school in pure mathematics, or anyone who enjoyed real analysis but wished it were cooler. To take this class, you should have already taken Math 321 but not Math 261, unless you get special permission from Owen.

Stats Classes

Stat 220: Intro to Data Science
Instructor: Katie St Clair
Time: 2a
Prerequisites: Stat 120, 230 or 250

This course will cover the computational side of statistics that is not typically taught in an intro or methodology focused course like regression modeling. Most of data you encountered in your first (or second, or third, ..) stats course were contained in small, tidy .csv files with rows denoting your cases and columns containing your variables. The only messiness to these data may have been some missing values (NAs). We will start this course in data science by learning how to extract information from data in its "natural" state, which is often unstructured, messy and complex. To do this, we will learn methods for manipulating and merging data in standard and non-standard formats, data with date, time, or geolocation variables, text processing and regular expressions, and scraping the web for data. To effectively communicate the information contained in these data, we will cover data visualization methods (or, as statisticians often call it, EDA) that go beyond a basic histogram or boxplot, including methods for creating interactive graphics. We may also cover some modern computationally-intensive statistical learning methods. We will primarily use the stat software R in this course.

Stat 230: Applied Regression Analysis
Instructor: Deepak Bastola
Time: 3a

Prerequisites: Statistics 120 or Statistics 250, Psychology 200, or AP Statistics Exam score of 4 or 5

Model-based thinking is at the core of statistics. In this course we will go beyond the realm of simple linear regression where we model a response variable as a linear function of an explanatory variable. We will see more advanced techniques for regression modeling, including models with many explanatory variables (multiple regression) or a categorical response (logistic regression). We will apply these techniques to a wide variety of data sets, with an emphasis on model building and checking and statistical writing. We will use the statistical software R extensively.

Stat 250: Introduction to Statistical Inference
Instructor: Laura Chihara
Time: 4a
Prerequisites: Math 240: Probability (formerly Math 265)

Statistics is the art and craft of studying data and understanding variability. Though mathematics (in particular, probability) governs the underlying theory, statistics is driven by applications to real problems. We will cover basic statistical inference as well as modern computational approaches, all in the context of investigating interesting questions that arise in scientific and public policy settings. We will use the software package R.

Stat 285: Statistical Consulting
Instructor: Andy Poppick
Time: Tuesday 10:10am-11:55am
Prerequisite: Stats 230 (formerly Math 245) and instructor permission

Students will apply their statistical knowledge by analyzing data problems solicited from the Northfield community. Students will also learn basic consulting skills, including communication and ethics.

Stat 320: Time Series Analysis
Instructor: Andy Poppick
Time: 5a
Prerequisite: Stat 230 and 250 (formerly Math 245 and 275)

Time series are data observed over time: think about things like a daily meteorological measurement, a quarterly economic index, or annual population size for species in a region. These kinds of data often have a special kind of dependence structure: recent observations are more informative about the present than are past observations. Statistical models that assume independence (such as those you learned in Applied Regression Analysis) do not suffice in this setting, which motivates the need for methods for modeling time series. This course will introduce you to two ways of thinking about time series -- the so-called “time-domain” and “frequency-domain” approaches -- and their connections to each other. The course will be a mix of theory and application, with an emphasis throughout on model-building and data exploration.

Comps Talks

Come support your classmates and friends at their comps talks next week! Group comps talks will take place 3:00 - 7:00pm in Olin 141 on both Tuesday, February 15 and Thursday, February 17. Take a look at what they'll be speaking about below, then be sure to stop by and support them while they show what they've learned; you're likely to learn a thing or two as well!

Tuesday, February 15 — Olin 141

Title: Rehumanizing Mathematics: Challenging Eurocentrism in the Undergraduate Mathematics Curriculum
Speakers: Ana Pina Marcelino, Stuart Kraabel, Collin Smith, and Sawyer Buhman
Time: 3 - 4pm

Abstract: Mathematics has a reputation of being the domain of white European men. Mathematicians such as Pythagoras, Fibonacci, and Pascal are lauded even though the mathematics they discovered had been known for centuries, if not millennia, beforehand in Africa, Asia, and Mesoamerica. There is danger in telling a single, culturally homogenous story of mathematical development. Students with minoritized identities may leave the subject because they do not see themselves or their communities represented in it, and omitting contributions from these communities perpetuates existing stereotypes of who can and can not be a mathematician.

Over the past two terms we have created a series of modules that can be implemented in the Carleton undergraduate mathematics curriculum. These modules are topic driven and contain culture of origin information, pedagogy notes, lesson plans, and example problems that instructors can use to challenge Eurocentrism in the classroom. In this talk we will discuss the motivations for this work, show how to access the modules, and dig into one of the modules as an example.

Title: Algebraic Topology and Discrete Morse Theory for Finite Spaces
Speakers: Elliot Heyman, Shira Julie, and Noah Pinkney
Time: 4 - 5pm

Abstract: In this presentation, we explore finite topological spaces and their algebraic structure. Given a finite topological space, we can construct an associated simplicial complex which contains information about the space. Likewise, we can start with a simplicial complex and construct a face poset, which is a finite topological space. After a brief discussion of homotopy theory, we show how information gained from investigating a given simplicial complex can provide insight into the associated finite space and vice versa. Lastly, we utilize discrete Morse theory to analyze these simplicial complexes and generate a method of simplifying an associated space while retaining information about the initial simplicial complex.

Title: Applications of Ensemble Analysis to Gerrymandering in Minnesota and Texas
Speakers: Oliver Calder, Antonia Ritter, Tom Patterson, Eva Airoldi, and Rebekah Stein
Time: 5 - 6pm

Abstract: Every ten years, all states must redraw a legislative districting map with the apportioned number of districts. Gerrymandering—methods by which political parties may attempt to gain electoral advantages by modifying district locations—is becoming more common in these redistricting processes. While maps can be, and have been, deemed illegal in court, there is no rigorous process for determining the extent to which gerrymandering is present in a map. In our analysis, we utilize the existing framework of ensemble analysis with some modifications to quantify the level of gerrymandering present in current and proposed maps for Minnesota and the current map for Texas. We generate a representative sample of all legal plans for the given state to identify whether the plan of interest is an outlier on a number of physical, demographic, and partisan metrics of gerrymandering. Additionally, through ensemble analysis, we discover whether underlying characteristics of a state—such as population distribution, geography, etc.—result in a natural bias towards a particular party.

We find Minnesota has a slight bias towards benefiting Republicans in our ensemble. For some metrics, the proposed plans appear within the expected range of scores; however, there is evidence of cracking and packing of Democrats, as well as Black and Hispanic voters. Our Texas results show a Republican bias. The 2010 Texas map has multiple scores that lean in favor of the Republican Party. There is some evidence of cracking and packing of Democrats, as well as Black and Hispanic voters in the 2010 map.

Title: Behind the Smoke: An Extreme Value Analysis of Air Pollution in Minnesota
Speakers: Jacob Flignor, Libby Nachreiner, Yicheng Shen, and Karen Wang
Time: 6 - 7pm

Abstract: Poor air quality is a major environmental health threat. Even short-term exposure to poor air quality -- such as during extreme pollution events -- can cause severe respiratory distress. While there have been significant decreases in Minnesota air pollution levels over the past 40 years, the summer of 2021 upset this trend with Hennepin County reporting the highest particulate measure in the past 20 years. This study aims to better understand the extreme values of pollutant concentration levels of sulfur dioxide (SO2) and fine inhalable particles (PM2.5) across three Minnesota counties as collected by the Environmental Protection Agency from 1980 to 2021. We employ extreme value analysis methods with temporal non-stationarity, including the Generalized Extreme Value (GEV) and Generalized Pareto (GP) distributions, to fit the pollutant data. The models find that SO2 levels have fallen substantially since 1980 in accordance with EPA policies regulating diesel fuel and coal power plants. As such, the low present-day baseline of SO2 levels amplifies the perceived magnitude of extreme events. Typical SO2 levels throughout the 1980s to 1990s would be considered one in a hundred year events if they occurred today. Additionally, no downward trend in PM2.5 levels was observed over the past 20 years -- an expected result given that PM2.5 has more varied sources and is therefore harder to regulate than SO2. However, models show a significant seasonal trend with peaks during winter months, revealing this past "summer of smoke" as particularly extreme.

Thursday, February 17 — Olin 141

Title: The Effect of Habitat Fragmentation on Plant Communities in a Spatially Implicit Grassland Model
Speakers: Mika Cooney, Ben Hafner, Shelby Johnson, and Sean Lee
Time: 3 - 4pm

Abstract: The spatially implicit Tilman-Levins ODE model helps to explain why so many plant species can coexist in grassland communities. This now-classic modeling framework assumes a trade-off between colonization and competition traits and predicts that habitat destruction can lead to long transient declines called "extinction debts". Despite its strengths, the Tilman-Levins model does not explicitly account for landscape scale or the spatial configuration of viable habitat, two factors that may be decisive for population viability. We propose modifications to the model that explicitly capture habitat geometry and the spatial pattern of seed dispersal for individual species. The modified model retains implicit space and is in fact mathematically equivalent to the Tilman-Levins model in the single species case. But its novel interpretation of a habitat destruction parameter better quantifies seed loss due to edge effects in fragmented habitats. We explore the implications of these modifications the context of Midwestern grasslands.

Title: Survival Analysis: An Application with Censored Head and Neck Cancer Data
Speakers: Sagal Ahmed, Henry Farnsworth, Karryn Leake, and Jeremiah Mackin-Alonzi
Time: 4 - 5pm

Abstract: Survival analysis is a branch of statistics focused on estimation for time to event data. Many areas, such as the medical field, engineering, or biology, rely on survival analysis to gain insight into what factors impact time to event. Using a dataset provided by McGill University containing 298 patients diagnosed with head and neck squamous cell carcinoma, we investigate what factors impact the survival prognosis of patients and determine the shape of their hazard rate. We employ non-parametric, semi-parametric, and parametric modeling techniques in our analysis. Namely, we use the Kaplan-Meier estimator to create survival curves, the Cox proportional hazard model to assess which factors significantly influence the hazard rate, and a log-normal model to find significant predictors of time to death. Our analysis was guided by four questions: Does survival prognosis differ depending on the hospital where a patient receives treatment? Does survival prognosis differ depending on the primary site of a patient's cancer? What is the shape of the hazard rate for head and neck cancer patients? What risk factors have the greatest effect on survival prognosis?

Title: A Long and Winding Road: A Foray into Planar Metric Graphs
Speakers: Soren DeHaan, Carl Tankersley, Iain McCay, and Henry Chapman
Time: 5 - 6pm

Abstract: Have you ever wondered what the fastest way is to walk from the CMC to LDC — through Olin? through Anderson? Around the outside if it's a nice day? In this talk we will formalize this problem as finding paths of minimal length through a graph, whose vertices are places you might want to go and whose edges are routes of various lengths connecting them. Given a list of starting points and destinations, we can build a matrix whose entries represent the shortest path lengths between them. What matrices arise this way? Given such a matrix, can we find the graph and edge lengths that gave rise to it? Good candidate graphs are the "critical, well-connected" planar graphs; we will describe a novel algorithm for enumerating and constructing them. We will also look at some other interpretations of edge-weighted graphs and the matrices one can construct from them, such as by adding the products of path weights, thinking of edges as strings that can be slack or taut, or by using imaginary edge weights to simulate "phantom" edges.

Title: Introduction to Lie algebras
Speakers: Juanito Zhang Yang and Owen Barnett
Time: 6 - 7pm

Abstract: A Lie algebra is a vector space with a bilinear form [—,—], called the Lie bracket, that satisfies the two properties [x,x] = 0 and [x,[y,z]] + [y, [z,x]] + [z, [x,y]] = 0. We will show some examples of Lie algebras, one of which is three-dimensional euclidean space with the cross product. Then we will discuss some important consequences of the two conditions. Finally, it will turn out that we can study Lie algebras through graphs. Specifically, we can use certain graphs called "Dynkin diagrams" to classify complex simple Lie algebras, which are Lie algebras over the complex numbers with no non-trivial ideals, and conclude that all complex simple Lie algebras, with five exceptions, belong to one of four families.

Job, Internship, & Other Opportunities

Camp Counselor - MathPath

MathPath, a residential summer program for students aged 11-14 (typically middle schoolers) who are exceptionally gifted in mathematics, is now hiring undergraduates for the Counselor position. Candidates for this position should be individuals who love math, interesting & accessible problems & puzzles; and love working with, spending time with, and caring for younger students (aged 11-14). For a detailed job description, please see: www.mathpath.org/public/files/CounselorJobDescription2022.pdf

We are looking for current undergraduate students (or recently graduated) who have declared a major in Mathematics (or Computer Science), or who have not yet declared a major but have serious interest in mathematics.

I invite you to apply or to send this information to friends or colleagues of yours who may be interested in the position. The deadline for first round selection to the position is February 15, 2022. For more detailed information about the application process see: www.mathpath.org/employment/counselors.

If you have any questions or would like more information about the position, please email Program Director April Verser at april.verser@mathpath.org.

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Problems of the Fortnight

To be acknowledged in the next *Gazette*, solutions to these problems should reach me by noon on Tuesday, February 22. It seems only fitting that with the 2/22/2022 due date, the number 2022 shows up again in the problems themselves ...

1. For the purposes of this problem, say a positive integer is *rich* if it has at least 2022 different prime factors. (So the smallest rich integer is the product $2 \cdot 3 \cdot 5 \cdot 7 \cdot \ \ldots \ \cdot p_{2022}$ of the first 2022 primes $p_1 = 2, p_2 = 3, p_3 = 5, \ldots$.) If there exists positive integers N and k such that *every* positive integer greater than k is the sum of at most N rich integers (that is, any sufficiently large integer is the sum of at most N rich integers), find (with proof) the smallest such N. If there is no such N, show why not.

2. Suppose we have an infinite family of equally spaced parallel lines in the x, y-plane. (One example of such a family would consist of the lines $y = 4x + n$, where n is an arbitrary integer.) Suppose we also know that the distance from the origin to the closest line in the family is $\frac{2}{5}$, that the distance from $(1, 0)$ to the closest line in the family is $\frac{1}{5}$, that the distance from $(2, 0)$ to the closest line in the family is $\frac{4}{5}$, and finally that the distance from $(3, 0)$ to the closest line in the family is 1. (Naturally, for different points the "closest line in the family" will not always be the same line.) Given all that,
   a) find the distance from $(2022, 0)$ to the closest line in the family;
   b) find all possibilities for the equations of the lines in the family.

Alas, no solutions to the problems posed January 28 have come in so far. They would still be welcome! Meanwhile, good luck on the new problems ...

- Mark Krusemeyer

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