Goodsell Gazette

Carleton College *November 19, 2021*

Northfield, MN 55057 *The newsletter for the Carleton mathematics and statistics community* *Vol. 40, No. 6*

Davis Projects for Peace Award

On 85 campuses across the country, 100 Projects for Peace will be funded this year. Prizes are $10,000 each and must be designed to promote world peace. Carleton can submit one proposal that will receive funding, and one alternate that might receive funding. Current Carleton undergraduates, including graduating seniors, are eligible. Group and individual proposals are possible.

The pre-application is due December 3; find it and more details at go.carleton.edu/peace.

Job, Internship, & Other Opportunities

National Center for Atmospheric Research - HAO - Summer Internships

The goal of the SIParCS program is to make a long-term, positive impact on the quality and diversity of the workforce needed to use and operate 21st century supercomputers. Graduate students and undergraduate students (who have completed their sophomore year by summer) gain significant hands-on experience in high-performance computing and related fields that use HPC for scientific discovery and modeling.

This program embeds students as summer interns in the Computational and Information Systems Laboratory, an organization within NCAR charged with provisioning supercomputing and data systems to the geosciences research community. The roles of service and research in CISL support NCAR's broad scientific mission of discovery in the atmospheric and related sciences.

See program specifics and apply on Handshake at carleton.joinhandshake.com/jobs/5636284.

Bruins-In-Genomics Summer Program - Summer Internship

Bruins-In-Genomics (B.I.G.) Summer Research Program is an 8-week full-time immersion program for undergraduates interested in learning how to read and analyze genes and genomes. Through this program students will have the opportunity to experience graduate-level coursework, and learn the latest cutting-edge research, tools and methods used by leading scientists to solve real-world problems.

Apply on Handshake at carleton.joinhandshake.com/jobs/5637595.

Statistics startup opportunity

Tate Fuller '25 is working with Jonah Docter-Loeb '25 on building an impact-focused social enterprise to enable consumers to understand the impacts (environmental, labor practices, political contributions, etc.) of the products they buy and companies they support. The associated app and website will also link consumer choice with direct political activism.

They are looking for students willing to help with this project–even just for an hour a week. Regardless of areas of expertise or experience, if you are even somewhat interested in the idea please fill out this form: docs.google.com/forms/d/e/1FAIpQLSe2Z8knpNEZs52_beuecS6j_VSqvzDYTdT2TY646ib1tv3beA/viewform.

Problems of the Fortnight

Because of winter break, the next *Gazette* will not appear until winter term, so solutions to the problems below that reach me any time in the next month and a half or so will be acknowledged in January.

1. Consider the "lattice points", that is, the points (m, n) with integer coordinates, in the plane. Let r be a number with $0 < r < 1$; for each lattice point, draw a circle of radius r with that lattice point at the center. It is not hard to see that after all these circles are drawn, every point in the plane that is not on one of the circles will be inside $0, 1, 2, 3$, or 4 of the circles; for instance, if $r = 0.8$, the point $(1/2, 1/2)$ will be inside the four circles centered at $(0, 0), (1, 0), (0, 1)$, and $(1, 1)$ (and not inside any of the other circles). Now color the points in the plane red that are inside an odd number (1 or 3) of the circles, while leaving the points that are inside an even number of the circles uncolored. As a function of r, what fraction of the plane will be colored red? (You will have to consider several cases, depending on the size of r. If it worries you, as well it might, that the area of the entire plane is infinite, you can rigorously define "fraction of the plane" by taking large finite subsets of the plane, say square regions with $-N \le x \le N, -N \le y \le N$, seeing what fraction of such a subset is red, and then taking the limit as $N \to \infty$. But because the whole pattern of circles is periodic, "taking the limit" doesn't really amount to much here.)

2. Let S be the set of real numbers that can be expressed as a difference of two 2021st roots of positive integers. That is, x is in S if and only if there exist positive integers m and n such that

$$x = \sqrt[2021]{m} - \sqrt[2021]{n}\,.$$

Certainly not every real number is in S; in fact, if you know about cardinalities of infinite sets, it's quite likely that you can show that the set S is countable, whereas the set $\mathbb{R}$ is uncountable. However, can every real number be approached arbitrarily closely by elements of S? That is, given a real number y and a positive tolerance ε, is there always a number x in S such that $|y - x| \le \varepsilon$? If so, show why; if not, give an example of a number y and a tolerance ε for which no such x exists.

Alas, although the last two weeks featured lingering fall colors, high winds, warm sunshine, snow, and other natural phenomena, no solutions arrived during that time. Solutions to the problems posed November 5 are still most welcome. Good luck finishing the term; have a great break, and see you (whether literally or not) in January!

- Mark Krusemeyer

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