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Math and Stats Comps Presentations!

Welcome to the Math and Stats Group and Independent Comps Presentations for Spring 2021! This term students are recording comps presentations that are viewable by a prerecorded video, available on our website. Please refrain from posting these videos to any other online resource. Students in both group and independent comps will be hosting a Q&A session by Zoom on May 18 and 20 at the times listed below. Watch your email for a Zoom invitation and for instructions for submitting questions for the Q&A. We hope you enjoy the comps presentations.

Tuesday, May 18

Title: Quantile Regression with Applications in Temperature and Agricultural Analysis
Speakers: Sarah Grier, Nate Isbell, Waleed Iftikhar, and Muyang Shi
Time: 3:40 - 4:00

Abstract: Across disciplines, researchers are often interested in gaining a deeper understanding of trends in the tails of a distribution or comparing trends across a range of quantiles in a distribution. Typical ordinary least squares (OLS) regression models do not permit this type of investigation and limit our understanding of a given data set to its mean trends. Quantile regression was developed by Koenker and Bassett in 1978 to solve this problem of needing to better understand trends in location, shape and skewness across quantiles. In this project we will investigate quantile regression as a method and utilize it to conduct case studies investigating the change in corn and rice yields in the United States and temperatures in Minnesota over the last 50 years. In the agricultural study, we discover that greater deviations in temperature from the average have adverse effects on growth rate of rice and corn yields and this adverse effect is much more pronounced at lower quantiles of percentage change in yield. In the temperature study across several locations in Minnesota, we see a consistent pattern of damping of the seasonal cycles but different responses as global temperature increases. Applying quantile regression, consequently, allows us to see a richer picture than what would be obtained through OLS models.

Title: Modular Forms and Number Theory
Speakers: Will Fletcher and Nick Rosen
Time: 4:40-5:00

Abstract: There are rich connections between number theory and certain complex-valued functions, the

most famous of which is probably the Riemann Zeta function. In this talk, we explore properties of another important class of functions with applications to number theory, modular forms. First, we build up several examples and ideas necessary to understand modular forms and their properties. Then we present some applications to number theory related to multiplicative arithmetic functions.

Title: Kreweras Complements of Noncrossing Tree Partitions
Speakers: Eliot Huh, Mei Knudson, Alec Wang, and James Yan
Time: 5:00 - 5:20

Abstract: Our research is focused on noncrossing tree partitions (NTPs), which are partitions of the interior vertices of a tree that can be represented by noncrossing curves connecting interior vertices. Given an NTP, we can apply an operation known as the Kreweras complement to produce a new NTP. However, there does not exist any algorithmic procedure for performing this operation. Our primary focus has been to develop and prove the validity of such an algorithm. We also discuss the connections of our work to quiver representation theory.

Title: The Topology of Achievement Sets Obtained Through Sums of Geometric Progression
Speakers: Santos Flores
Time: 5:20 - 5:40

Abstract: What would happen if we decide to omit certain terms of a sequence? How will that change the shape and convergence of them? Through the use of achievement set we can choose to omit certain terms and consider this outcome using geometric progressions. We will discuss different properties and topological significance behind the omission problem of terms of sequences. We get to see whether the omissions of terms of sequences create a bound of achievable numbers or potentially something else like the Cantor set.

Thursday, May 20

Title: Topology of Partially Ordered Graph Configuration Spaces
Speakers: Pierce McDonnell and Hiromichi Ueda
Time: 4:00 - 4:20

Abstract: Imagine that we have a set of robots moving along tracks in a factory, where the robots are points and the tracks are a graph. The "graph configuration space" is what we call the collection of all the ways that we can move the robots around the graph, without causing any collisions. In our project, we discuss the properties of these spaces and explain how to describe them more simply. In particular, we look at graph configurations for which the robots are labeled using some number of distinct colors. For example, imagine that we have two blue robots, A and B, and one red robot C; we cannot tell A from B, but we can distinguish them from C. We explore the characteristics of these "partially ordered" configurations, and offer a proposal for how to concisely describe their graph configuration spaces.

Title: Just Plane Confused: The Absent-Minded Passenger Problem
Speakers: Jackson Wahl
Time: 4:20 - 4:40

Abstract: What happens when the first passenger to board an airplane can't remember where to sit? Do most people sit in their seats, or does chaos reign? A recent paper by Norbert Henze and Gunter Last focuses on the absent-minded passenger problem, in which airline passengers boarding a plane in a fixed order. The first passenger picks a random seat; the rest sit in their own seat, unless it is occupied, in which case they pick a random seat. I examine the expectation and full distribution of wrongly seated passengers, as well as an extension to multiple absent-minded passengers.

Title: Solving the Word Problem in Hyperbolic Groups
Speakers: Erik Carlson, Honus Frohlich, Nick Brusilow
Time: 4:40 - 5:00

Abstract: In any finitely generated group, the task of determining whether or not some string of generators represents the identity element of the group is called the word problem. In the general case, this task cannot be done in a finite amount of time. We present a solution to the word problem in a geometrically-defined class of groups called hyperbolic groups.

The solution to this lies in the realm of Geometric Group Theory. Geometric Group Theory is a branch of Abstract Algebra wherein properties of groups, about which perhaps little is known, are inferred from the actions of those groups on geometric objects, or as we investigate, spaces. We study groups which act in a specially-defined way on a class of spaces called hyperbolic spaces; such groups are called hyperbolic groups. In this talk we will demonstrate how the characteristics of hyperbolic groups, determined by their actions on hyperbolic spaces, allow us to construct an algorithm for solving the word problem in these groups.

Title: Transient Dynamics in Ecological ODE Models: Plant Populations under Habitat Destruction
Speakers: Qianzi Li, Lucy Reece, Sophie Schafer, Muyang Shi
Time: 5:00 - 5:20

Abstract: Habitat destruction causes biodiversity loss by imposing "extinction debts" on plant species, causing slow transient declines in populations that are fated to crash. Certain spatially implicit ODE models developed by Tilman et al. illuminate the influence of habitat destruction on the balance between species's colonization and local mortality processes. Scientists at Cedar Creek Ecosystem Science Reserve recently started a long-term field experiment to test whether seed addition can rescue grassland species following habitat destruction. We report on our collaboration with the scientists using both traditional bifurcation analysis and flow-kick modeling. Under Tilman's modeling framework, we compute expected timescales of species loss without intervention and assess the possibility of compensating for habitat deficits by enhanced colonization. We find that boosting colonization is a viable restoration strategy in theory, whereas sustained rescue of populations facing an extinction debt may require perpetual intervention.

Comps Announcement Meeting

On Tuesday, May 25 from 4:00 - 5:00pm we will be holding a virtual Zoom meeting to announce the comps topics for next year. All math and stat juniors should plan to attend. Math or stat majors who are currently listed as the class of 2023 but who plan to graduate in 2022 should let us know that you will need to do comps next year. If you have questions, you can reach out to Sue Jandro or Andy Poppick.

Upcoming Events

Week 8

Tuesday May 18, 3:40 - 5:40pm
Comps Q&A Sessions - Zoom

Thursday May 20, 4:00 - 5:20pm
Comps Q&A Sessions - Zoom

Week 9

Tuesday May 25, 4:00 - 5:00pm
Comps Announcement Meeting - Zoom

Problems of the Fortnight

To be acknowledged in the next *Gazette*, solutions to the problems below should reach me by noon on Tuesday, May 25.

1. As motivation for this problem, here's a standard result from geometry: A convex quadrilateral (four-sided figure) $ABCD$ can be inscribed in a circle (that is, all four vertices lie on the same circle) if and only if the sum of the (opposite) angles at A and at C is $180°$. Now for the problem: Let $ABCDEF$ be a convex hexagon, with the vertices $A, B, C, \ldots$ in counterclockwise order.

   a) Suppose $ABCDEF$ can be inscribed in a circle. Does it follow that the sum of the three angles at A, at C, and at E is $360°$? Why, or why not?
   b) Suppose the sum of the three angles at A, at C, and at E is $360°$. Does it follow that $ABCDEF$ can be inscribed in a circle? Why, or why not?

2. Call a complex number z an *integer root* if there is a positive integer n such that z^n is an integer. For example, $(1+i)/\sqrt[4]{2}$ is an integer root, because the fourth power of this number is

$$\frac{(1+i)^4}{2} = \frac{(2i)^2}{2} = -2\,.$$

Now let S be the set of all complex numbers w for which there is some infinite sequence $z_1, z_2, z_3, \ldots$ of integer roots whose limit is w. (For instance, S certainly contains all integer roots, because any integer root z is the limit of the particular sequence $z, z, z, \ldots$ - the sleepy sequence, perhaps?) Find the set S, and show why your answer is correct. (That may sound a bit vague, but there is a straightforward description of S that makes no mention of integer roots or infinite sequences.)

There were several inquiries about the problems posed April 30, and I may be able to report more solutions to them in the next *Gazette*; for now, Juanito Zhang Yang and Sebastian Vander Ploeg Fallon both solved problems 1 and 2a, and Sebastian solved 2b as well. Both Juan and Sebastian should arrange with Sue Jandro to pick up a B.B.O.P. item. Good luck on the new problems, and enjoy the gorgeous spring weather while it lasts ...

- Mark Krusemeyer

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