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Carleton College *February 19, 2021*

Northfield, MN 55057 *The newsletter for the Carleton mathematics and statistics community* *Vol. 39, No. 09*

Math and Stats Group Comps Presentations

Welcome to the Math and Stats Group Comps Presentations for Winter 2021! This term students are recording comps presentations that are viewable by a prerecorded video. The prerecorded videos will be viewable by noon on Friday, February 19 on our website. Please refrain from posting these videos to any other online resource. Group comps students will be hosting a Q&A session by Zoom on February 23 and 25 at the times listed below. Watch your email for a Zoom invitation for viewing and instructions for submitting questions for the Q&A. We hope you enjoy the comps presentations.

Tuesday, February 23

Title: Dedekind Domains, Prime Factorization and Elliptic Class Groups
Speakers: Jack Heinzel, Daniel Kleber, Matt Mendiola
Time: 4:30 - 4:50 pm

Abstract: We study the properties of unique factorization of prime ideals in Dedekind domains, and the class group of Dedekind domains. We ask the question of what abelian groups can be constructed as the class group of a Dedekind domain, and find that every abelian group is the class group of some Dedekind domain. In this talk, we consider the countable case. We use the theory of elliptic curves to generate a Dedekind domain with a prescribed countable abelian class group.

Title: Bernoulli Numbers and the Class Group of Cyclotomic Fields
Speakers: Marco Bommarito, Marcella Manivel, Gavin Peng, and Marguerite Shaya
Time: 5:00 - 5:20 pm

Abstract: In this talk, we explore the Bernoulli numbers: a sequence of rational numbers with connections to combinatorics, complex analysis, algebraic number theory, etc. Importantly, Bernoulli numbers may be used to define the B-regular and B-irregular primes, which have historical connections to investigations of Fermat's Last Theorem. In 1850, Ernst Kummer related B-irregularity of primes to the class numbers of cyclotomic fields. Over the course of the talk, we build towards an understanding of this connection and its mathematical significance.

Thursday, February 25

Title: Predicting COVID-19 using State-Space SIR Models
Speakers: Travis Brown, Vincent Gu, Marko Jurkovich, Andrew Vance
Time: 4:00 - 4:20 pm

Abstract: What will COVID cases look like in one month? How many people have already been infected? What would be the efficacy of another lockdown? In our talk, we will give an introduction to statistical disease modeling using state-space SIR (Susceptible-Infected-Removed) models that aim to answer these questions. We then test two Bayesian models' predictive accuracy in the pandemic using Israel as a case study.

Title: Northcott's Theorem: an adventure in preperiodic points of rational functions
Speakers: Evan David, Ian Klein, Ben Richardson, Sameer Swarup
Time: 4:30 - 4:50 pm

Abstract: What happens when you repeatedly apply a function to a rational number? Some numbers eventually loop back on themselves; Northcott's theorem states that there are only finitely many of these apreperiodic numbers for a given rational function. We introduce a notion of height to provide a surprising proof of Northcott's theorem. During this proof, we make an emergency foray into algebra in the form of homogeneous polynomials and polynomial rings. We also introduce the canonical height, which measures how close a number is to being preperiodic.

Title: Network Analysis in R: Two Case Studies
Speakers: Cindy Guo, Jenna Korobova, Erika Mino, Matt Zacharski, Daniel Zin
Time: 5:00 - 5:20 pm

Abstract: Social media has become an increasingly common way for people around the globe to engage in discourse and seek information. Data that captures relationships between users on various platforms including followers, hashtags, likes, and retweets is conducive to representation in the form of a network. The structure and characteristics of these networks can be analyzed using combinations of mathematical and statistical approaches for graphs. In particular, data from Twitter is a rich source of information about users, topics, and communities that can be analyzed using the statistical methods in R. Our Twitter investigation considers two case studies: (i) a comparison pro- and anti-mask networks in various states during the COVID-19 pandemic and (ii) an analysis influential users and their characteristics in a network of retweets about the death of George Floyd. In the first case study, we found similar structural characteristics in both networks though the anti-mask communities were larger, and we found that the edge-betweenness and infomap algorithms for community detection yielded similar results. Finally, in the second case study, we found that shared sentiment amongst users was the strongest predictor of retweets, and that @YourAnonCentral and @AttorneyCrump were the most retweeted (influential) users.

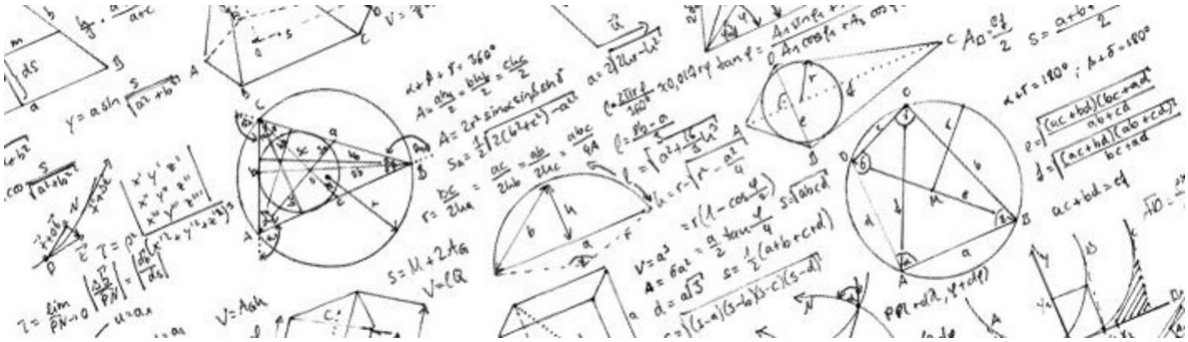
Title: Virtual Edge Detection and Bone Measurements
Speakers: Maddie Kyhl, Abby Loe, Charlotte Clapham, Bat-Orgil Batjargal
Time: 5:30 - 5:50 pm

Abstract: Bones can inform our understanding of the past and make predictions for the future. Information is encoded in characteristics of the bone, and applying mathematical tools to bone analysis can provide robust and interesting insights. The AMAAZE group at the University of Minnesota specializes in these questions--particularly, questions about broken bones. One of their current projects is working to reassemble broken bone fragments and classify broken bones by how they were broken. In order to answer these questions and more, they need to collect information on the angle of the bone break, which can be measured using their virtual

goniometer tool--a virtual tool capable of measuring these angles on 3D bone scans. This process introduces interesting mathematical questions. How does the curvature of the bone's surfaces relate to this angle measurement--can we find a relationship to inform the measurement process? Also, how do we actually detect where the bone was broken inside this virtual tool? With our research, we hope to understand and provide new insight to AMAAZE's measurement tool and larger mission.

What's the Math and Stats Department Teaching Next Term?

Have you checked your registration number yet? Made a list of classes you're hoping to take next year? Let the course descriptions below guide you into an adventurous new term with the Carleton Department of Mathematics and Statistics! There's something for everybody, from abstract algebra to applied regression â€" find out more below.



Math Courses

Math 236: Mathematical Structures
Instructor: Alex Barrios
Time: 2a and 3a, online (2a section sophomore priority)
Prerequisite: Math 232 or permission from the instructor

How do we prove mathematical statements? How do we even think of possible statements, and what makes us suspect that a particular statement may be true? There are no easy, general answers. Mathematics is a complex subject, with a great variety of living and growing branches, and with deep roots that tap into the wisdom of many generations. Still, if you've ever wondered "How did anyone come up with that?", or "How can you really be sure of that?", about some mathematical result, taking this course may help dispel some of the mystery. We'll explore various concepts, especially from set theory, that are indispensable for most areas of advanced mathematics, and we'll spend considerable time developing theorem-proving and problem-solving skills. Along the way we'll take a new and closer look at some old friends, such as functions and relations: What are they really? In the final part of the course we'll use functions to compare "sizes" of various infinite sets. For example, we'll see that despite appearances, there are not any "more" rational numbers than there are integers; on the other hand, there are "more" real numbers than rational numbers. If you're considering a math major, taking this course should help you decide; also, "Structures" is a prerequisite for the majority of upper-level math courses.

Math 241: Ordinary Differential Equations
Instructor: Kate Meyer
Time: 3a, online, synchronous
Prerequisite: Math 232 or instructor permission

Differential equations are a fundamental language used by mathematicians, scientists and engineers to understand and describe processes involving continuous change. In this course, we will study differential equations from both a practical and theoretical point of view. Our focus will be on developing differential equation models from natural laws and exploring the mathematical ideas that arise within these models. Examples may include mechanical vibrations, lasers, insect outbreaks, competition and cooperation of species, and more! The science will stay at an elementary level; our focus will be the mathematical ideas that arise in these models.

Math 261: Functions of a Complex Variable
Instructor: Mark Krusemeyer
Time: 5a, online, synchronous
Prerequisite: Math 210 or Math 211

What happens to calculus when you replace the real variable x by the complex variable $z = x + iy$ and real-valued functions $y = f(x)$ by complex-valued functions $w = f(z)$? Surprising and wonderful things! The statement "f is differentiable" becomes more powerful, while the idea of integration becomes more flexible – you can now integrate along various paths in the complex plane. Not only does this lead to much beautiful mathematics, but, and this might surprise you, a lot of this material can be applied to *real* (pun intended) mathematical and physical problems in which nary an imaginary number occurs. For instance, we should see how to compute some improper integrals of functions that don't have an antiderivative in closed form, such as

$$\int_0^\infty \frac{\cos x}{x^2+1}\,dx\,,$$

and if time permits we'll see how to apply complex transformations to solve problems of heat flow in the plane. On the other hand, we'll sketch a proof of the "Fundamental Theorem of Algebra", which states that any nonconstant polynomial with complex coefficients has a (complex) root – and thus any such polynomial factors completely into linear factors. A more theoretical version of this course, with real analysis as a prerequisite, is offered in alternate years; if you are planning to go to graduate school in pure mathematics and you aren't a senior yet, you should probably wait for Math 361 next year.

Math 331: Real Analysis II
Instructor: Rafe Jones
Time: 3a
Prerequisites: Math 321 or instructor permission

How can we talk about the size of a set of real numbers? How can we integrate as many real-valued functions as possible? The answers to these questions turn out to be closely related, and take us beyond notions of size from Mathematical Structures (countable vs uncountable) and notions of integration from calculus (the Riemann integral). By the end of the course, we will be able to â€œmeasureâ€ a vast array of subsets of the real numbers, and integrate functions that leave the Riemann integral gasping for breath. The course includes the Lebesgue measure, the Lebesgue integral, an introduction to general measure theory, and Banach and Hilbert spaces. Not only is this your chance to "integrate" your knowledge of functions, it is also an opportunity

to better your understanding of the legal interchange of limit operations. Rumor has it that the Cantor set may also make an appearance.

The flavor of the course will be similar to a graduate-level course in analysis. If there is a possibility that you are headed for graduate school in mathematics or a related field, this course comes highly recommended.

Math 333: Combinatorial Theory
Instructor: Eric Egge
Time: 5a, hybrid
Prerequisites: Math 236 or permission of the instructor

I looked in my sock drawer this morning and saw a jumbled collection of 36 socks, consisting of 6 socks in each of 6 colors. Each sock had a single letter stitched on it, and within each color, each of the letters J, S, E, A, K, and R appeared exactly once. In how many ways, I wondered, can I match my 36 socks into 18 pairs, so that both socks in each pair have the same color? Dreaming of spring, when I don't need socks any more, can I display my socks in a 6 by 6 square, with no color or letter repeated in any row or column? Then my son wandered into the room. He has his own definition of which pairs of socks match, which seems to have nothing to do with letters or colors. If I close my eyes and start removing socks from the drawer, how many must I remove before I am guaranteed to have three socks in which each pair matches (according to my son) or three socks in which no pair matches?

If you, like me, are intrigued (or tormented) by questions like these, then combinatorics might be the right course for you. We'll study techniques for showing certain arrangements of things exist (or don't), and techniques for counting these arrangements when they do exist. Some of these counting techniques involve playing with power series, without worrying about convergence! We'll pay particular attention to counting sequences (like the Catalan numbers and the partition numbers) which have especially remarkable properties, and we'll use our counting techniques to prove some of the myriad identities involving the numbers in Pascal's triangle. We'll also make periodic forays into graph theory, and near the end of the course we'll see a ``proof'' of the four color theorem. Although this proof will have a gap (which I'll ask you to find), we will also learn about some of the key ideas in the actual proof of this famous result.

The content for this course will be delivered on videos, but as long as it's safe to do so, we will also meet in person once each week. At most of these meetings we will work in small groups, sometimes on interesting problems related to the material we're covering in the videos, sometimes on combinatorial ideas we don't have time for in the videos, and perhaps even on an open problem or two.

You don't need any previous knowledge of combinatorics to take this course, just experience with the proof techniques from structures, the ability to multiply polynomials, a willingness to try new and strange problems, and a sense of adventure.

Math 341: Partial Differential Equations
Instructor: Rob Thompson
Time: 3a, online
Prerequisite: Math 241

About 200 years ago, Jean Baptiste Fourier studied the way that heat moves through a flat metal plate via a partial differential equation called the heat equation. Trying to describe his observations mathematically, he did a seemingly simple thing: he expressed the heat distribution as a sum of sines and cosines (a "Fourier series"). Expressing the complicated behavior of heat in terms of simpler functions gave Fourier powerful insight into the behavior of the heat equation. Fourier's idea revolutionized pure and applied mathematics. In this course, we'll learn the fundamentals of partial differential equations and make a tour of Fourier's revolution. We'll examine various interesting PDE (including the heat equation) and their applications to wave

propagation, heat conduction, elastic equilibrium, and more. We'll also develop ideas from Fourier analysis as needed to access information about the solutions to the PDE we study. Feel free to contact me (rthompson) with any questions!

Math 342: Abstract Algebra I
Instructor: MurphyKate Montee
Time: 4a, online
Prerequisite: Math 236 or instructor permission

Abstract Algebra is the study of algebraic structures. It grew out of generalizing number systems, like the integers and the reals. We'll start with groups, the axiomatization of symmetry. From the study of these objects, we'll be able to gain a deep understanding of relationships between polynomials, permutations, matrices, geometric objects like polygons, and more. We'll also apply our knowledge of groups to the study of number systems we're already familiar with, and extensions of those systems (eg complex numbers). We'll then expand our view: we'll start looking at rings and fields - objects with more structure than groups, and less flexibility. While this all sounds pretty abstract, the systems and theorems developed in this course will appear throughout mathematics; once you start seeing groups, they're everywhere!

Math 395: Introduction to Analytic Number Theory
Instructor: Caroline Turnage-Butterbaugh
Time: 2a, online with synchronous class sessions
Prerequisite: Math 312 or instructor permission

Euclid proved that there are infinitely many primes. If N is a large number, how many primes are there less than or equal to N? Can we describe this count asymptotically, as N grows arbitrarily large? The theorem that describes this behavior, the Prime Number Theorem, was eventually proved in 1896 using techniques from complex analysis. Well before this proof (and without using complex analytic techniques) much progress was made towards this endeavor. In this class, we'll explore tools and techniques which are analytic in nature to solve problems related to the integers and, more specifically, the primes. Without any experience in (real or complex) analysis assumed, we will prove results towards the Prime Number Theorem and culminate the course in a proof of Dirichlet's Theorem on Primes in Arithmetic Progressions.

This course is open to all students who have taken Math 312; contact me if you are interested in the course but have not taken Math 312. If you have taken more advanced courses (such as real analysis, complex analysis, and/or Galois theory), you will have the opportunity to apply such experience via personalized final projects.

Statistics Courses

Stat 220: Intro to Data Science
Instructor: Katie St. Clair
Time: 2a, online
Prerequisites: Stat 120, 230 or 250 (a Carleton Stats course)

This course will cover the computational side of statistics that is not typically taught in an intro or methodology focused course like regression modeling. Most of data you encountered in your first (or second, or third, ...) stats course were contained in small, tidy .csv files with rows denoting your cases and columns containing your variables. The only messiness to these data may have been some missing values (NAs). We will start this course in data science by learning how to extract information from data in its "natural" state,

which is often unstructured, messy and complex. To do this, we will learn methods for manipulating and merging data in standard and non-standard formats, data with date, time, or geolocation variables, text processing and regular expressions, and scraping the web for data. To effectively communicate the information contained in these data, we will cover data visualization methods (or, as statisticians often call it, EDA) that go beyond a basic histograms or boxplots, including methods for creating interactive graphics. We may also cover some modern computationally-intensive statistical learning methods. We will primarily use the R programming language in this course.

Stat 230: Applied Regression Analysis
Instructor: Laura Chihara
Time: 1a, online (sophomore priority)
Prerequisite: Stat 120 (AP statistics 4/5) or Stat 250

On the night of January 27, 1986, engineers at Morton Thiokol teleconferenced with engineers and managers at the Marshall Space Flight Center and Kennedy Space Center to determine whether it was too cold (31 F) to launch space shuttle Challenger. Data from previous flights seemed to suggest that temperature had an effect on the integrity of the O-ring seals on the booster rockets, but the final recommendation was to launch the Challenger on schedule. Could a statistical analysis of the pre-accident data predicted the catastrophic failure of the shuttle? In this class, we will investigate the Challenger data and in general, learn statistical model building and model checking techniques. We will use the software package R to aid in the modeling.

Stat 250: Introduction to Statistical Inference
Instructor: Laura Chihara
Time: 3a, face-to-face
Prerequisite: Math 240 Probability

Statistics is the art and craft of studying data and understanding variability. Though mathematics (in particular, probability) governs the underlying theory, statistics is driven by applications to real problems. We will cover basic statistical inference as well as modern computational approaches, all in the context of investigating interesting questions that arise in scientific and public policy settings. We will use the software package R.

Stat 285: Statistical Consulting
Instructor: Katie St. Clair
Time: Tuesday's 10:20-12:05 pm, online
Prerequisite: Stat 230 (Math 245) and instructor permission

Students will work on data analysis projects solicited from the local community. We will also cover the fundamentals of being a statistical consultant, including matters of professionalism, ethics and communication.

Stat 320: Time Series Analysis
Instructor: Andy Poppick
Time: 2a, online
Prerequisites: Stats 230 and 250 (Math 245 and 275)

Time series are data observed over time: think about things like a daily meteorological measurement, a quarterly economic index, or annual population size for species in a region. These kinds of data often have a special kind of dependence structure: recent observations are more informative about the present than are past observations. Statistical models that assume independence (such as those you learned in Applied Regression Analysis) do not suffice in this setting, which motivates the need for methods for modeling time series. This course will introduce you to two ways of thinking about time series -- the so-called "time-domain" and "frequency-domain" approaches -- and their connections to each other. The course will be a mix of theory

and application, with an emphasis throughout on model-building and data exploration.

Job, Internship, & Other Opportunities

Bonds of Union - Ascend Teaching and Innovation (ATI) Fellowship

We are hoping to recruit motivated and thoughtful students and recent graduates to apply for our 2021 Ascend Teaching and Innovation (ATI) Fellowship. This is a two-year fellowship based in Cincinnati that begins in August 2021. Selected fellows serve as Learning Coaches, working together one-on-one with students in grades 3-5 at a low-performing public school on individualized, â€œlow-floor, high-ceilingâ€ math tasks that foster a growth mindset and support social-emotional development. Here is a link to more information, and a brief video that explains what we do and what makes Ascend so special: http://www.bondsofunion.org/ascend/.

The next rolling application deadline is March 1. Find the application at the link above.

Southern Teachers Agency

K-12 private and independent schools around the South are searching for great teachers to join their school communities for next year. Schools plan to have in-person classes in the fall, and we have over a hundred math openings for next academic year.

Our schools seek teachers with subject-matter expertise who are enthusiastic about working with children and teenagers what is important is a gift for connecting with and caring for kids. Certification is typically not required (except in elementary grades and specialty areas).

Southern Teachers has matched great candidates with schools in the South for over a century, taking the time to interview candidates, and know the person, not just the resumÃ©. Our services are free for our candidates! The sooner you apply, the more opportunities you will have to pursue as you search for the right school community for you.

Apply now at SouthernTeachers.com.

Snowflake - Data Marketplace Challenge

Show us how you drive value for your business with third-party data!

Enter the Snowflake Data Marketplace Challenge for a chance to win a virtual meet-and-greet with Snowflake Co-Founder & President of Products, Benoit Dageville; a complimentary Snowflake SnowPro Certification course; marketing exposure; Snowflake swag; and more!

Registration is now open at www.snowflake.com/data-marketplace-challenge/.

Minnesota Center for Financial and Actuarial Mathematics - "How I Became a Quant" Virtual Panel

Show us how you drive value for your business with third-party data!

Enter the Snowflake Data Marketplace Challenge for a chance to win a virtual meet-and-greet with Snowflake Co-Founder & President of Products, Benoit Dageville; a complimentary Snowflake SnowPro Certification course; marketing exposure; Snowflake swag; and more!

Registration is now open at www.snowflake.com/data-marketplace-challenge/.

Problems of the Fortnight

This is a special issue, so there is no problem of the fortnight this week, but feel free to take a look at problems from past issues online.

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Problems of the Fortnight: ***Mark Krusemeyer***
Web & Subscriptions: ***Sue Jandro***