# Goodsell Gazette

*Carleton College* *September 29, 2017*

*Northfield, MN 55057* *The newsletter for the Carleton mathematics and statistics community* *Vol. 36, No. 02*


## Meet Your New Professors!

**Owen Biesel**

Owen, originally from Washington, moved around a lot with his father in the Navy spending most of his childhood in Virginia and Connecticut. He received his undergraduate degree from the University of Washington, spent a year in Cambridge, before completing graduate school at Princeton. Following graduation, he spent three years in the Netherlands completing his postdoc and learning Dutch along the way. Owen is interested in many aspects of math including algebraic geometry. His research specifically focuses on the generalization of the ideas underlying ellipses and hyperbolas and other geometric shapes defined by algebraic equations. He studies generalizations of the quadratic formula to other number systems. During his free time, he likes to knit (he may even be wearing a sweater that he has knit), read, and listen to audio books on his drive to work. Owen is excited about teaching at Carleton with teachers who are truly focused on teaching and supporting their students and students who are equally passionate about learning. Owen is teaching Calc I and will be teaching Calc II, Linear Algebra, and supervising a comps group this winter and spring.

**Coming Next Week:** Meet our new Professor Adam Loy!

## Summer Activities Lunch

Are you wondering how you might spend next summer? Do you get hungry at lunchtime? Come to CMC 209 on Tuesday, October 3 at noon to satisfy both these needs! There will be free pizza, and Maya Banks, Annie Shapiro, Logan Crowl, Emily Kaegi, and Qimeng Yu will discuss their experiences with research and internships, and answer your questions.

## Problem Solving Group

If you have always really enjoyed the problem-solving aspect to your classes, then the problem-solving

group is just for you. Come join us in CMC 328 from 4:30-5:30 on Wednesdays, where we will work on solving some fun and challenging math problems together. All are welcome.

Rafe Jones is hosting the sessions. You can contact him for more information at (rfjones@carleton.edu).

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## Putnam Registration Time is Here!

Fall Term is just getting underway, but it's already time to register for this year's William Lowell Putnam Mathematical Competition. As many of you know, the "Putnam" is a challenging exam focusing on mathematical insight and ingenuity. Typically, several thousand undergraduates across the United States and Canada participate, and the median score is usually less than 10 out of a possible 120. Whether you've taken the exam before or are considering taking it for the first time, you'll probably enjoy getting experience with past Putnam problems (and learning some new problem-solving strategies) at our weekly problem-solving group, which meets every week on Wednesday, from 4:30 to 5:30 in CMC 328.

This year the Putnam will be held on Saturday, December 2. That's during our winter break, but we'll gladly make arrangements for you to take the Putnam at another college or university. If you'd like to sign up, contact Rafe Jones in person or via email (rfjones). For more information, see the bulletin board outside Math Skills, where a brochure will soon be posted, or talk to Rafe. Act soon! Although the Putnam is still more than two months away, we have to submit a participant list soon, so your deadline for signing up is October 1 this coming Sunday!

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## SWiMS Mentoring

SWiMS (Society of Women in Math and Statistics) offers a mentoring program for freshmen and sophomore female and non-binary students who are interested in math and/or statistics. We pair interested students with current majors to offer a more personal look into our math and stats community. If you are interested in signing up for this program, please fill out this survey https://goo.gl/forms/9oNuw0NtKBM0lGHT2. For more information, please talk to Liz Sattler (lsattler@carleton.edu).

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## Northfield Undergraduate Mathematics Symposium (NUMS)

**Date: Wednesday, October 4**
**Location: Regents 210, St. Olaf**

Each year Carleton and St. Olaf students work on a variety of interesting research problems in mathematics, both here in Northfield and around the country. Several of these students will be sharing the work they did this past summer at the 2017 Northfield Undergraduate Mathematics Symposium. Please join us for as many of the talks as you can attend, as well as for dinner. These talks all promise to be fascinating in their own right, but each one also counts for half of one of the eight talks junior and senior math and stats majors need to attend to satisfy their lecture attendance requirement. Below is a schedule for the symposium, followed by the titles and abstracts for the talks.

**Schedule of Talks**

3:40 - 4:00 pm On the properties of kth-Order Fibonacci-like polynomials
Katherine Arneson, St. Olaf College

4:05 - 4:25 pm A model of hierarchy emergence in complex networks
Maya Banks, Carleton College

4:30 - 4:50 pm Invariantization of finite difference approximations on differential equations
Spencer Eanes and Shane Kosieradzki, St. Olaf College

4:55 - 5:15 pm Automated jigsaw puzzle assembly and invariant signatures
Peter Illig and Qimeng Yu, Carleton

5:20 - 5:55 pm Dinner (will be provided)

6:00 - 6:20 pm Factorizations of k-Nonnegative Matrices
Neeraja Kulkarni, Carleton College

6:25 - 6:45 pm Refined Inertia for Sign Patterns
Derek DeBlieck and Deepak Shah, St. Olaf College

6:50 - 7:10 pm Mutational signature analysis with the Indian buffet process
Sophia Gunn, Carleton College

7:15 - 7:35 pm Topological data analysis on various applications
So Mang Han and Xiaojun Zheng, St. Olaf College

**Titles and Abstracts**

Title: On the properties of kth-Order Fibonacci-like polynomials
Speaker: Katherine Arneson, St. Olaf College

The well-studied Fibonacci polynomials are described by $F_n(x) = xF_{n-1}(x) + F_{n-2}(x)$ with $F_0 = 1$ and $F_1 = x$. We extend this recursive polynomial sequence to $G_n^{(k)}(x) = xG_{n-1}^{(k)}(x) + G_{n-k}^{(k)}(x)$ and further to $H_n^{(k)}(x) = xH_{n-1}^{(k)}(x)-H_{n-k}^{(k)}(x)$ with initial conditions $G_0^{(k)} = G_1^{(k)} = ... = G_{k-1}^{(k)} = 1$ and $H_0^{(k)} = H_1^{(k)} = ... = H_{k-1}^{(k)} = 1$. When $k = 2$, the $G_n^{(k)}$ polynomials are the Fibonacci polynomials with altered initial conditions. When $k = 3$ and $x = 1$, the $G_n^{(k)}$ polynomials describe Narayana's Cow Sequence. In this talk, we present a closed form for these polynomials from which we derive a class of significant integer sequences. For example, we find a sequence that describes a population that takes 5 months to mature and reproduces every 3 months, which has been submitted to the On-Line Encyclopedia of Integer Sequences. We also study some interesting analytic properties of these polynomials, especially concerning the behavior of their roots. We establish that the minimal real roots of $G_n^{(k)}(x)$ converge uniformly to some number $r^{(k)}$ in the open interval from -3 to -1, and the maximal real roots of $H_n^{(k)}(x)$ converge uniformly to 2. MATLAB simulations show that the roots of $G_n^{(3)}$ are rational for only $n = 3, 5, 6, 10$, and 19, and those of $H_n^{(3)}$ are rational for only $n = 3$ and 12. Computer-assisted curve fitting using MATLAB suggests that the relative rates of convergence of the minimal real roots of $G_n^{(k)}$ and the maximal real roots of $H_n^{(k)}$ are of exponential order.

Title: A model of hierarchy emergence in complex networks
Speaker: Maya Banks

Given a directed network, we can infer a hierarchy by extracting a ranking of the vertices from the adjacency matrix of the network. In systems such as animal social groups or faculty hiring networks, these rankings

allow us to better understand the relative status of different individuals. While there exist different ranking models that allow us to observe hierarchy in established systems, we are left with many questions about how hierarchical structure evolves in networks in the first place. In real networks where the edge set changes over time, we expect that hierarchy in the network affects the formation of new edges. At the same time, the hierarchy that we observe is extracted from the edges currently present in the network. We explore a dynamical model of hierarchy emergence in networks that produces a strongly tiered hierarchical structure, using a physics-inspired method of ranking vertices.

Title: Invariantization of Finite Difference Approximations on Differential Equations
Speaker: Spencer Eanes and Shane Kosieradzki

To increase accuracy of numerical solutions to differential equations, the equations' inherent geometric properties, called Lie symmetries, can be exploited. These symmetries encompass a change of variables that preserves solutions. They are an integral part of the equations that exhibit them, but are not considered by traditional finite difference approximations. We can parameterize these symmetries based on a set of normalization equations, the result of which is known as moving frame. Using this moving frame allows the finite difference scheme to inherit the differential equations' symmetries, thus constructing a corresponding invariant scheme. We will show that invariantization increases accuracy of multistep schemes on ordinary differential equations, as well as the Crank-Nicolson method applied to Burgers equation.

Title: Automated jigsaw puzzle assembly and invariant signatures
Speaker: Peter Illig and Qimeng Yu

"Turning and turning in the widening gyre
The falcon cannot hear the falconer;
Things fall apart; the centre cannot hold;
Mere anarchy is loosed upon the world..."
-William B. Yeats

When things fall apart, who can put them back together? In this talk, we will present computational methods to reassemble blank jigsaw puzzles, relying on shape alone to determine assembly information. Using a measurement of shape known as an invariant signature, we discuss the process of finding matching puzzle pieces and algorithms for aggregating these matches into an assembled puzzle.

Title: Factorizations of k-Nonnegative Matrices
Speaker: Neeraja Kulkarni

A matrix is said to be totally nonnegative if the determinant of every square submatrix in it is nonnegative. More generally, a matrix is said to be k-nonnegative if the determinant of every square submatrix of size at most k is nonnegative. Factorizations of totally nonnegative and k-nonnegative matrices have long attracted attention because of their many applications in data analysis and their interesting connections with topology. In particular, the set of invertible totally nonnegative matrices can be partitioned, based on factorizations, into cells that form a CW-complex. We give a set of generators into which k-nonnegative matrices can be factored in two special cases: k = n - 1 for any invertible matrices, and k = n - 2 for triangular matrices with 1s on the diagonal. We prove that these sets of matrices can also be partitioned into cells that are homeomorphic to open balls and prove some results about the closure of these cells. This work was done with Joe Suk of Stony Brook University and Ewin Tang of the University of Texas, Austin; and supervised by Sunita Chepuri, Pavlo Pylyavskyy and Vic Reiner of the University of Minnesota, Twin Cities.

Title: Refined Inertia for Sign Patterns

Speaker: Derek DeBlieck and Deepak Shah

Motivated primarily by the Inverse Eigenvalue Problem, the mathematical community has invested large amounts of time and energy into the study of the relation between matrices and their eigenvalues. We contribute to that study by examining which 3x3 sign patterns, matrices with non-numeric 0/+/- entries, allow certain types of eigenvalues. In particular, we are interested in patterns that allow refined inertia S3 = {(0, 3, 0, 0),(0, 2, 1, 0),(1, 2, 0, 0)}. In other words, we seek patterns that have one realization with all negative real-part eigenvalues, another with all negative real-part except for one zero eigenvalue, and a third with all negative real-part except for one positive real-part eigenvalue. The presence of this property is of particular interest, as it signals the presence of a saddle-node bifurcation in the study of dynamical systems. We classify all 19,683 3x3 sign patterns and determine which of these patterns allow this specific set of eigenvalues. We also present a theorem for the extension of our work beyond 3x3 patterns.

Title: Mutational Signature Analysis with the Indian Buffet Process
Speaker: Sophie Gunn

In the field of cancer genomics, mutational signatures are used to characterize the somatic mutations that occur over the lifetime of a tumor. Each mutational signature corresponds to a specific mutational process, like UV damage or the loss of certain DNA repair mechanisms. Discovering these signatures from a cohort of cancer patients can be formulated as a machine learning problem; the overall collection of mutations for a given patient can be thought of as a linear combination of latent mutational signatures. Thus, the goal of mutational signature analysis is to decide the mutational signatures present in a given cohort and how many mutations each signature contributes to each patient. Mutational signatures are often found using Bayesian non-negative matrix factorization (NMF). This method works very well; it determines the number of signatures present in a cohort and is also quite fast. However, traditional Bayesian NMF assigns every patient every mutational signature, even when it is unlikely that the patient has a specific signature. My talk will focus on my work this summer adding in the Indian Buffet Process, a stochastic process defining a probability distribution over sparse binary matrices, to the current NMF framework to determine whether a given signature is present in the patient or not.

Title: Topological Data Analysis on Various Applications
Speaker: So Mang Han and Xiaojun Zheng

Topological data analysis (TDA) is a collection of the methods that finds the shape of data. TDA is especially useful to extract information of high dimensional and noisy data, which can be challenging for a geometric approach to analyze the structure of data. Persistent Homology is a main tool for TDA to bridge ideas between geometry and topology. Using RIVET, an interactive visualization software for two-parameter persistent homology, the structure of a network map of Game of Thrones was analyzed. RIVET detects structures of sub-graphs of the final network graph and provides detailed information that users can understand what is happening at each level. We also analyzed the structure of a point cloud produced from a semantic analysis of Simple English Wikipedia articles. We used RIVET to distinguish the Wikipedia point cloud from a point cloud of similar random vectors. We compared the topological similarity of Wikipedia articles for major cities using semantic distance and geographic distance between the cities. In addition to analyzing the RIVET plots, we applied statistical tests to the topological differences between the data sets to confirm our conclusions.

# Math and Stats Colloquium: Mind-Bending Paradoxes and the Possibility of Changing Your Mind

**Dave Kung, St. Mary's College of Maryland**
**Tuesday, October 10, 7:30pm, Olin 141**

Great riddles and paradoxes have a long and illustrious history, serving as both tests and games for intellectual thinkers across the globe. Passed through the halls of academia and examined in-depth by scholars, students, and amateurs alike, these mind-benders have brought frustration and joy to those seeking intellectual challenges. Choosing to confront these conundrums, we put ourselves in that special moment when we acknowledge that what we previous thought conflicts with some new piece of evidence. Those are the moments - rare and precious - when we might actually change our minds! Topics will range from the philosophical to the statistical, from physics to psychology, all from the perspective of a mathematician. Prepare to have your mind bent - and maybe even changed.

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# Winter Break Research Funding Opportunity

The Kolenkow-Reitz Fund provides student stipend and travel support for Carleton students working with non-Carleton science and math faculty at another institution during winter break. Awards fund student stipends ($440/week for full-time work) for up to 3 weeks during winter break and can include expenses for travel, lodging, and meals. No award will exceed $1500. Note that students must work full-time in order to qualify. Carleton students are eligible to apply for this funding. Before applying, students should have already contacted and discussed the nature and timing of their project with the person they are planning to work as well as a faculty member at Carleton who can vouch for the project. Please note that previously funded students through the Kolenkow-Reitz Fund (winter break or summer) are less likely to get funded, but are still eligible to apply. The application deadline is Thursday, October 12 at 5:00 p.m. Please contact Jennifer Wolff (jwolff@carleton.edu) if you have any questions. More details are available in the application form, which can be found at https://apps.carleton.edu/mathscience/faculty/studentresearchaway.

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# Iowa State University

Are you considering Grad School? Iowa State offers many wonderful and interdisciplinary programs. For more information, visit: https://www.grad-college.iastate.edu/academics/programs/approgram s.php. Iowa State is also offering students the opportunity to find faculty members that match their research interests and can potentially work with by visiting: http://scholars.proquest.com/gallery/IASTATE/results?text=.

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# Boston College, Lynch School of Education

Are you considering a career in education? Well, pursuing a graduate degree at Boston College might be just for you. Boston College is looking for the next cohort of students to join their graduate school. For more information, visit their webside: http://www.bc.edu/bc-web/schools/lsoe/about.html.

# Upcoming Events

**Week 4**, Tuesday, October 3, 12:00 - 1:00pm
Summer Activities Panel - CMC 209

**Week 4**, Tuesday, October 3, 4:00 - 5:00pm
Tea - Math Skills Center

**Week 4**, Wednesday, October 4, 3:40 - 7:40pm
NUMS - St. Olaf

**Week 5,** Tuesday, October 10, 7:30 - 8:30pm
Dave Kung Colloquium - Olin 141

**Week 6**, Tuesday, October 17, 4:00 - 5:00pm

Tea - Math Skills Center

# Job & Internship Opportunities

**Horizons Fellowship**

The Horizons Fellowship supports 150 outstanding university students in their pursuit to become leaders in technology. Our tuition-free programs provide immersive software engineering and web/mobile development courses geared towards high-achieving college students. Students need not have a computer science background! Our curriculum, developed by ex-Salesforce and Optimizely engineers alongside PhD candidates in computer science, is designed to teach students how to build web, mobile, and desktop applications. Since we first began a few years ago, Horizons has taken over 200 high-achieving university students through our programs. We've increased our scholarship pool dramatically and have been able to accept students from all walks of life and backgrounds into our fully-funded fellowship programs. Horizons students have gone on to receive offers from firms such as Google, Slack, Yelp, Amazon, BCG, Visa, J.P. Morgan, and more. Applications are accepted on a rolling basis. To learn more, visit: www.joinhorizons.com.

**Federal Reserve Bank, Research Analyst**

Research Analysts play an integral role in both the policy and research functions of the Research and Statistics Group. Economists at the Federal Reserve Bank, whose specialties include banking and payment systems, capital markets, international economics, macroeconomics, and microeconomics, work closely with Research Analysts. Upon leaving the Fed, Research Analysts who choose to apply to graduate school are consistently accepted by top programs; others pursue a wide variety of public and private sector opportunities. They are seeking candidates who have records of superior scholarship and academic curiosity. Research Analysts usually come from strong economics, policy, mathematics, or computer science backgrounds, though a major in one of these fields is not a necessity. Successful candidates often have previous research experience, and many are considering careers in economic research, public policy, or other related fields. In addition, they are seeking candidates from a wide range of backgrounds that are typically underrepresented in economics. The Federal Reserve Bank is committed to recruiting a diverse

cohort of Research Analysts each year. Thus, many students with varying experiences and backgrounds are encouraged to apply. Applications are being accepted now, on a rolling basis, at www.newyorkfed.org/careers. It is recommended that candidates apply by October 6th.

**Stanford Opportunity Lab, Predoctoral Fellowship**

The Stanford Opportunity Lab is looking to hire several full-time pre-doctoral fellows for the academic year 2018-19 to work with their research group on the Equality of Opportunity Project. The overarching mission of the project is to develop scalable policy solutions that will empower families throughout the United States to rise out of poverty and achieve better life outcomes. This mission is pursued primarily by conducting rigorous, scientific research that harnesses the power of "big data" using an interdisciplinary approach. Positions are available in Stanford CA, Cambridge MA, Providence RI, and Washington D.C. under the direction of Professors Chetty, Friedman, and Hendren. The fellowship will include a variety of tasks that provide preparation for graduate school, such as analyzing data, developing statistical models, creating presentations, editing, and writing research papers. Applicants should be completing (or have completed) a Bachelors and have strong quantitative and programming skills. Candidates with research experience are strongly preferred, especially those with experience in Stata, SAS, R, Python, or Hadoop. Candidates need not be Economics majors, though they should have a strong interest in and some experience with Economics. Candidates would ideally begin on June 1, 2018, and work for two years before applying to graduate school in Economics or another quantitative social science. To apply, please: 1) Submit a formal application through Stanford (Job Number: 76029) 2) Upload your resume, cover letter (optional), resume, transcripts, and a writing sample through the group's online form. Questions can be directed to Sarah Merchant and Jamie Gracie at chettylab@gmail.com. Applications received by October 31, 2017, will be eligible for the first round. First-round decisions will be announced around November 15, 2017. For more information, visit: http://www.equality-of-opportunity.org/jobs/predoc_app.pdf.

# Problems of the Fortnight

To be acknowledged in the next *Gazette*, solutions to the problems below should reach me by noon on Tuesday, October 10.

1. Any "great circle" on a sphere (a circle whose center is at the center of the sphere, such as the equator on the earth, if only the earth were really a sphere) will divide the sphere into two hemispheres (in the case of the equator, Northern and Southern). For the purposes of this problem, consider the great circle itself to be part of both hemispheres so formed. Find the largest number, say $n$, such that whenever $n$ points are chosen on the surface of a sphere, it is possible to draw a great circle on that sphere such that all $n$ points end up in the same hemisphere. (Of course, you should explain why your number $n$ is correct.)

2. Consider an $n \times n$ matrix (square array), all of whose entries are 0 and 1, for example

$$\begin{matrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}.$$

If we look at the $n$ rows of this matrix and interpret them as binary representations of integers (in the example, $n = 4$, and the integers are $1101_2 = 8 + 4 + 1 = 13, 1001_2 = 9$, $0001_2 = 1, 0000_2 = 0$), then add up those integers, we get an integer $R$ (in the example, $R = 13+9+1+0 = 23$). If, instead, we interpret the *columns* of the matrix as binary numbers and we add those numbers, we get another integer $C$ (in the example, the columns yield the integers $1100_2 = 12, 1000_2 = 8, 0000_2 = 0, 1110_2 = 14$ and so $C = 12 + 8 + 0 + 14 = 34$). Finally, we can subtract $R$ from $C$ to get a single integer $D = C - R$ (in the example, $D = 11$). Now for the problem:

   a) As a function of $n$, what is the largest possible value of $D$ (for all possible $n \times n$ matrices with entries chosen from $\{0, 1\}$)?
   b) Still as a function of $n$, how many different values of $D$ can occur? How do you know?

The first problem posed September 15 was solved by "Möbius Quip"; the first part was also solved by prospective student Ethan Rojek. The second problem was solved by Jack Heinzel, who should stop by CMC 217 to collect a B.B.O.P. item. Well done, all! [There was also an attempted solution to the second problem that I haven't quite deciphered yet, but that doesn't seem to get to the heart of the matter.] Good luck on the new problems ...

- Mark Krusemeyer

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*Problems of the Fortnight:* ***Mark Krusemeyer***
*Web & Subscriptions:* ***Sue Jandro***