Computational complexity theory seeks to classify mathematical problems according to the efficiency with which they can be solved by algorithms. An algorithm is said to run in polynomial-time if, given an input of n bits, it halts after at most $O(n^k)$ steps, for a constant k. For most problems of interest, polynomial-time algorithms are substantially faster than brute-force approaches, so traditional complexity theory uses polynomial runtime as a proxy for efficiency. Seminal work by Cook, Levin, and Karp in the 1970s established a large class of problems which cannot be solved in polynomial-time assuming the widely believed P≠NP conjecture [1, 2, 3]. This framework says nothing, however, about the relative hardness of problems for which polynomial-time algorithms exist. As researchers work with increasingly large datasets, ostensibly small differences in asymptotic complexity can result in enormous differences in actual runtime. In practice, even algorithms that run in time $O(n^2)$ are infeasible on very large inputs.

Fine-grained complexity seeks to refine this theory by establishing runtime lower bounds for problems that are solvable in polynomial time. This involves proving that the existence of a certain sub-cubic- or sub-quadratic-time algorithm would violate one of several widely believed hardness assumptions. This line of research has yielded strong results for numerous ubiquitous problems, including the calculation of string edit-distance [4], approximate graph diameter [5], and the colinearity of three points in a plane [6]. Researchers had previously spent decades searching for fast algorithms for many such problems. Results from fine-grained complexity provide strong evidence no such algorithms exist.

Dr. Virginia Vassilevska Williams at MIT has helped pioneer this line of research. If awarded the Paglia Fellowship, I will work with Dr. Vassilevska Williams's group to study the fine-grained complexity of sensitivity problems. In a sensitivity problem, queries are made to a data structure that undergoes small, bounded alterations with respect to its initial state. For many such problems, it is an open question whether a preprocessing step can attain algorithms that run faster than recomputing a new query from scratch after each update. Previous work by Dr. Vassilevska Williams and collaborators explored the fine-grained complexity of several problems from graph theory in a sensitivity setting [7]. During this fellowship, I will seek to build upon this research.

My work will consist of devising proofs for reductions based on believable fine-grained hardness assumptions. Previous fine-grained complexity research has explored problems across a diverse set of fields including linear algebra, graph theory, and discrete geometry. Hardness reductions intrinsically tie together distinct mathematical structures, so my research will involve learning a wide array of cutting-edge mathematical techniques spanning multiple fields. I will do this work alongside PhD students and other researchers working with Dr. Vassilevska Williams. Given the collaborative nature of math research, this work requires making connections that can only be found at a larger research university. Two years in Dr. Vassilevska Williams's group will provide me with ample opportunities to engage in research at the forefront of complexity theory.

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